

Planck's Solution to the Ultraviolet Catastrophe an Early 20th century crisis in physics

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The born of quantum physics

In the realm of classical physics, energy is considered as a continuous quantity, flowing smoothly and seamlessly without bound. In the late 19th century two prominent physicists, Lord Rayleigh and James Jeans, using classical physics, attempted to explain the electromagnetic radiation emitted by an idealized object (a black body) that absorbs all incident radiation. Their theory, the Rayleigh-Jeans law¹, predicted that the intensity of emitted radiation would continuously increase indefinitely with increasing frequency. However, experimental observations revealed a stark discrepancy: the intensity peaked at certain frequency and then sharply decreased at higher frequencies. This mismatched phenomenon was labeled and known as the "Ultraviolet Catastrophe".

Planck's Quantum Hypothesis

To address and resolve the ultraviolet catastrophe, Max Planck introduced the revolutionary concept of energy quantization in 1900. But Planck himself initially viewed his quantum hypothesis as a mathematical trick to adapt the formula for the blackbody radiation. At the beginning, he didn't fully embrace the idea of quantized energy as a fundamental reality, and many physicists were opposed to accept the radical notion of quantized energy, as it challenged the well-established principles of classical physics at the time. In his hypothesis Planck proposed that energy is not emitted or absorbed continuously, but in discrete packets, or quanta, now known as photons, and it was proportional to the frequency of the photon, as described by the following equation:

$$E = h\nu$$

where:

(E) is the energy of the photon, (h) is Planck's constant and (ν) is the frequency of the radiation

By 1905, Einstein's explanation of the photoelectric effect provided strong evidence for the existence of photons, solidifying the concept of quantized energy. Additionally, the development of matrix mechanics by Werner Heisenberg and wave mechanics by Erwin Schrödinger in the mid-1920s, provided a more robust and mathematically rigorous foundation for quantum mechanics.

¹ Formula for Rayleigh-Jeans' law: $B(\nu, T) = (2\nu^2 k T) / c^2$

The 1920s saw a surge in experimental and theoretical work that further validated the principles of quantum mechanics, and the development of quantum field theory and the discovery of electron spin further solidified the acceptance of quantization. The transition from the classical Rayleigh-Jeans law to Planck's quantum theory represents a pivotal moment in the history of physics. It demonstrated the limitations of classical physics in describing the behavior of matter and energy at the atomic and subatomic level. A good analogy to understand the quantization process is a staircase. To ascend or descend the stairs, one must take discrete steps of a fixed height. It is not possible to advance a fraction of a step; one must take a whole step.

The way radiation is absorbed in a black body is determined by the energy distribution of its vibration modes. This distribution has a peak maximum at a specific frequency, depending on the black body's temperature. Hotter temperatures have sharper and taller peaks located at higher frequencies, while lower temperatures have broader and lower peaks at lower frequencies. The shift of the peak at different temperatures is described by Wien's law. At higher and lower frequencies from a given temperature, the absorption distribution decreases rapidly due to the mismatch between photon energy and the available energy in the vibration modes of the system. The quantization process led to Planck's law, which accurately predicted the spectral distribution of black body radiation across all frequencies, and is described by the following equation:

$$B(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

where:

$B(\nu, T)$ is the spectral radiance, (ν) is the frequency of the radiation, (k) is the Boltzmann constant, (T) is the absolute temperature and (c) is the speed of light.

Beyond Black Body Radiation

The implications of Planck's quantum theory were profound. It marked the birth of quantum mechanics, revolutionizing our understanding of the atomic and subatomic world. It transitioned the classical view of energy transfer from a continuous flow, as wrongly predicted by the Rayleigh-Jeans law, into a discrete step process, where the energy of the absorbed photon must precisely match the energy difference between the initial and final energy states of the atom or molecule within the black body. The observed energy property from different experiments could not be explained with classical physics, as it is the case for:

- The atomic spectra of atoms where discrete lines are observed due to the transition of electrons from one energy level to another
- The photoelectric effect, where photons from a beam of light directed onto a metal surface eject electrons. Classical physics predicted that the energy of the ejected electrons should increase with the intensity of the light; however, experiments showed

that the energy of the electrons depended only on the frequency of the light, a fact that could only be explained by assuming that light energy is quantized.

- Classical physics predicted that the specific heat capacity of a solid should increase linearly without bound with temperature. However, at low temperatures, the specific heat capacity of solids decreases rapidly, approaching zero as the temperature approaches absolute zero.

In contrast, Planck's quantized and discrete energy has proven time and time again to be the right approach for the above and many other examples.

The following diagram displays the Black-body radiation as a function of wavelength at various temperatures, as described by Planck's law. A connection between the maximum intensity emissions for different temperatures shows the displacement described by the Wien's law. Additionally, the red dotted line approximately shows what the Rayleigh-Jean law wrongly predicted at a given temperature, allowing intensity go to infinite at higher frequencies (lower wavelengths), when applying classical physics.

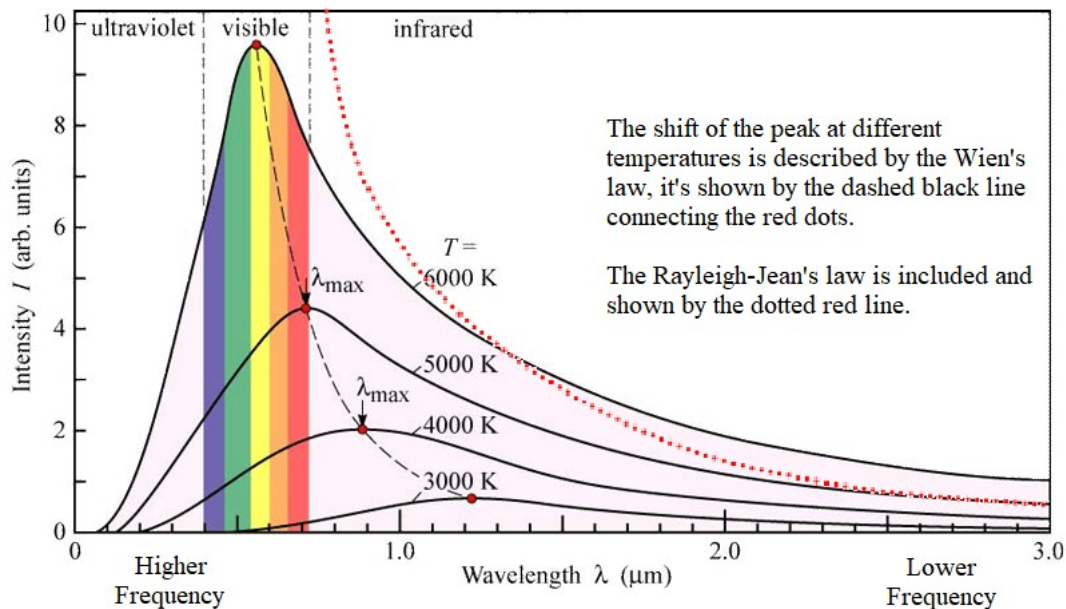


Diagram showing the Planck's law for some temperatures and the corresponding Wien's displacement